

# A COMPARISON OF PARTITIONED AND NON-PARTITIONED MATRIX SOLUTIONS TO COUPLED FINITE ELEMENT-INTEGRAL EQUATIONS

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The coupled finite element-integral equation technique is developed for solving radiation and scattering problems involving inhomogeneous objects with large dimensions in terms of wavelength. This method incorporates an exact integral equation outside an arbitrary surface, very near the object and circumscribing it, instead of an approximate absorbing boundary condition on a surface much farther away from the object, which is typically employed in finite element solutions (T. Cwik, et al., IEEE-APS Symposium, June 1993). As shown in the figure below, the complete matrix equation resulting from the coupled approach involves a large sparse submatrix  $K$  which is due to the finite-element mesh within a body of revolution (BOR) boundary, relatively small and banded submatrices  $Z_m, Z_j$  describing the integral equation, and two rectangular strip submatrices,  $C$  and  $C^+$  which represent the coupling between the finite element and the integral equation at the boundary. The unknowns are the magnetic field inside the BOR and the fictitious surface currents  $M$  and  $J$  on the BOR.

The system may be solved in three ways. In a single-step non-partitioned solution, an iterative solver is used. A complex non-Hermitian quasi minimal residue (QMR) algorithm is applied. This is simple and straight-forward but has to be repeated for multiple right-hand sides. In a two-step approach, the unknown magnetic field  $H$ , is eliminated by applying a complex but symmetric QMR to the symmetric  $K$  matrix. The problem is then reduced to a system for  $M$  and  $J$  currents and is solved by a direct method (i.e., LU factorization) which can handle multiple right-hand sides, while the operation on  $K$  is performed only once. Finally, in a three-step partitioned solution, the  $M$  or  $J$  unknown is eliminated by yet another direct solver. Then, the final matrix equation for the remaining unknown current,  $J$  or  $M$ , can be broken down into individual submatrix equations for each Fourier mode of the current, each of which can be solved for by applying a direct solver. The results obtained by the above methods are compared and the relative speed, memory requirement, and accuracy issues are discussed.

$$\begin{bmatrix} K & C & 0 \\ CT & 0 & Z_0 \\ 0 & Z_m & Z_j \end{bmatrix} \begin{bmatrix} H \\ M \\ J \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V \end{bmatrix}$$

